Topology

B. Math. II

Semestral Examination

Instructions: All questions carry equal marks.

1. Define *linear continuum*. Let X be an ordered set with order topology. Show that if X is connected, then X must be a linear continuum.

2. Let X and Y be topological spaces and $\mathcal{A}$ be a collection of basic open sets in $X \times Y$ such that no finite subcollection of $\mathcal{A}$ covers $X \times Y$. If X is compact, show that there exists a point $x \in X$ such that no finite subcollection of $\mathcal{A}$ covers $\{x\} \times Y$.

3. State the *second countability axiom* for topological spaces. If $\mathbb{R}_l$ denotes the set of real numbers with lower limit topology, then prove that it is not metrizable.

4. Define a normal topological space. Prove that a space X is normal if and only if given a closed set A and an open set U containing A in X, there is an open set V containing A such that $\overline{V} \subset U$.

5. State Urysohn Lemma. Let X be a normal space and A, B be two closed subsets of X. Prove that there is a continous function $f : X \to [0, 1]$ such that $f^{-1}(\{0\}) = A$ and $f^{-1}(\{1\}) = B$, if and only if A and B are disjoint G_δ sets in X. (Recall that a set is called G_δ if it is the intersection of countable collection of open sets).